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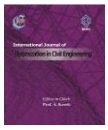

A TRUST-REGION SEQUENTIAL QUADRATIC PROGRAMMING WITH NEW SIMPLE FILTER AS AN EFFICIENT AND ROBUST FIRST-ORDER RELIABILITY METHOD

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ABSTRACT

The real-world applications addressing the nonlinear functions of multiple variables could be implicitly assessed through structural reliability analysis. This study establishes an efficient algorithm for resolving highly nonlinear structural reliability problems. To this end, first a numerical nonlinear optimization algorithm with a new simple filter is defined to locate and estimate the most probable point in the standard normal space and the subsequent reliability index with a fast convergence rate. The problem is solved by using a modified trust-region sequential quadratic programming approach that evaluates step direction and tunes step size through a linearized procedure. Then, the probability expectation method is implemented to eliminate the linearization error. The new applications of the proposed method could overcome high nonlinearity of the limit state function and improve the accuracy of the final result, in good agreement with the Monte Carlo sampling results. The proposed algorithm robustness is comparatively shown in various numerical benchmark examples via well-established classes of the first-order reliability methods. The results demonstrate the successive performance of the proposed method in capturing an accurate reliability index with higher convergence rate and competitive effectiveness compared with the other first-order methods.

Keywords: Structural reliability; expectation; failure probability; nonlinear optimization.

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1. INTRODUCTION

Multifold integrations with high computational costs are required for accurately determining

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the probability of failure in reliability analysis [1–3]. Alternative integration methods include sampling methods, specifically Monte Carlo Sampling (MCS), where thousands of simulations are needed to determine the probability of failure [4–6]. The First-Order Reliability Method (FORM) has been developed to overcome the computational cost in recent decades. FORM is a fundamental reliability analysis method recommended by the international Joint Committee of Structural Safety (JCSS) for calculating reliability based design optimization [7–9]. FORM investigates standard normal space to find the Most Probable Point (MPP) or design point. MPP is a point on the limit state surface with a minimum distance from the origin [10,11], which is also called reliability index. Rackwitz and Fiessler further modified Hasofer and Lind’s method [12] by considering non-normal random variables [13]. The method is known as HLRF, one of the first versions of FORM. If the limit state surface is flat, HLRF immediately converges. However, if it is highly nonlinear in the standard normal space, HLRF may encounter unexpected errors such as truncation, bifurcation, periodic oscillation, and chaotic behavior. Zhang and Kiureghian considered the appropriate choice of the step size with merit function monitoring [14]. In other words, using the Armijo rule, the researchers optimized the step size to develop their improved HLRF as iHLRF. The Stability Transformation Method (STM) is an algorithm driven by the chaos control theory introduced by Yang to handle the non-convergence issue of HLRF [15]. Roudak et al. proposed the adaptive chaos control to reduce the number of iterations in STM [16]. Meng et al. further proposed the directional stability transformation method (DSTM) to improve the stability of HLRF [17]. Additionally, Gong and Yi employed a Finite Step Length (FSL) parameter, another approach to compute failure probability in the direction of gradient vector of limit state function to estimate the reliability index [18]. Keshtegar introduced two advanced versions of FSL that involve the conjugate search direction and adaptive finite-step length as CFSL and AFSL [19,20]. There are some methods based on metaheuristic algorithms which implement different procedure close to sampling method [21,22].

Numerical nonlinear optimization algorithm is another approach to finding design points in the structural reliability problem. Well-known instances in this regard include Gradient Projection (GP), Augmented Lagrangian Method (ALM), and Sequential Quadratic Programming (SQP) [23–26]. SQP, a popular robust optimization method, is a gradient-based method to solve inequality and equality constraint problems. Indeed, the initial idea of employing the first-order reliability method was triggered by the SQP algorithm.

In optimization problems with many variables, SQP is coupled with the trust-region method to facilitate the convergence required for coming up with the final solution. This successful combination of the trust-region and sequential quadratic programming is known as the trust-region sequential quadratic programming (trust-region SQP) which uses the second and first derivatives of the objective function and constraint, respectively [27–29]. Trust-region SQP has already been applied to a number of pure optimization problems; however, its application to reliability-based engineering optimization is limited. The aerodynamic wing geometry optimization presented by Joongki et al. is one of the successful implementations of the trust-region SQP is in the reliability analysis [30]. The inequality type of trust-region SQP is the algorithm the researchers used to control the feasible domains of problem constraints as they function properly for addressing large-scale problems.

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This study intends to propose the equality constraint type of trust-region SQP with a simple filter for reliability analysis that is proper for non-large-scale problems with highly nonlinearity limit state function and the problems with single minimum point, not multiple design points. In other words, structural reliability problem consists of a simple objective function and nonlinear constraint. Accordingly, the constraint of the reliability problem is the major part of the optimization problem. On the other hand, the merit function which is used in the main version of the trust-region SQP, equipped with second-order correction of the final result, does not significantly improve the accuracy and robustness of structural reliability problems. Therefore, replacing the old filter with a new simple one eliminates some berries to the fast convergence of the algorithm. However, simple and efficient techniques are also elaborated to improve the accuracy of the proposed algorithm. Consequently, the proposed combination with applied changes leads to the desired results.
The rest of the article is structured as follow: Section 2 reveals the theoretical details of the target method for ensuring reliability analysis. A brief review of the probability expectation method is provided in Section 3. The proposed method is then evaluated in comparison with other studies concentrating on a set of benchmark examples with different properties in Section 4. It is followed by discussion in Section 5 and conclusion in Section 6.

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2. TRUST-REGION SEQUENTIAL QUADRATIC PROGRAMMING

A reliability problem can be considered a nonlinear equality constrained optimization problem which is shown in the Eq. (1) [31].

$$\begin{cases} \min & f\left(x\right)=0.5\left\|x\right\|^2 \\ \text{subject to} & G\left(x\right)=0 \end{cases} \tag{1}$$

where f is the objective function; G is the equality constraint or limit state function; and x represents the decision variables in optimization field or vector of all random variables (x_i) in the standard normal space in the reliability of structure field. The objective function f and the equality constraint G are supposed to represent continuous second-order nonlinear differentiable functions. Eq. (2) presents the Lagrangian function corresponding to the optimization problem of Eq. (1).

$$L\left(x,\lambda\right)=f\left(x\right)+\lambda^T G\left(x\right) \tag{2}$$

where λ is the Lagrangian coefficient. Trust-region SQP is one of the most robustness methods for solving these equality-constrained optimization problems. In this regard, it is necessary to define the trust-region SQP type of the optimization problem by modifying Eq. (1) as Eq. (3) included the quadratic objective function, linear constraint function, trust-region constraint.

$$\begin{cases} \min & \nabla f\left(x_k\right)^T d_k + \dfrac{1}{2} d^T B_k d_k \\ \textit{subject to} & G\left(x_k\right) + \nabla G\left(x_k\right)^T d_k = 0 \ \textit{and} \ \left\| d_k \right\| \le \Delta_k \end{cases} \tag{3}$$

where $\nabla f(x_k)$ is the gradient vector of objective function respect to x_k; d_k is the step direction vector; Δ_k is the current trust-region radius; and the matrix B_k represents Hessian for the Lagrangian equation of Eq. (2), which can be calculated by approximation methods such as Broyden-Fletcher-Goldfarb-Shanno (BFGS) method [32]. The optimization problem in Eq. (3) can be decomposed to two sub-problems. The first one is a normal sub-problem as expressed by Eq. (4).

$$\begin{cases} \min & \left\| G\left(x_k\right) + \nabla G\left(x_k\right)^T d_N \right\| \\ \textit{subject to} & \left\| d_N \right\| \le \zeta \Delta_k \end{cases} \tag{4}$$

where ζ is a constant between 0 and 1 with a recommended value of 0.8 [31]. Denote the solution to Eq. (4) by d_N. When the solution of Eq. (4) is achieved, the residual vector, r_k, is obtained, $r_k=G(x_k)+ \nabla G(x_k)^T d_N$. The vector d_N in Eq. (4) is the normal part of the step direction vector obtained by solving the normal sub-problem. The *dogleg* method is an approximate solution to the optimization problem Eq. (4) [33].

The second stage is to solve the tangential sub-problem that is shown in Eq. (5).

$$\begin{cases} \min & \left(\nabla f\left(x_k\right) + B_k d_N\right)^T d_T + \dfrac{1}{2} d_T^T B_k d_T \\ \textit{subject to} & \nabla G\left(x_k\right)^T d_T = 0 \ \text{ and } \ \left\| d_T \right\| \le \sqrt{\Delta_k^2 - \left\| d_N \right\|^2} \end{cases} \tag{5}$$

The projected conjugate gradient (PCG), suitable for solving the equality constrained problems (EQP) [34], is one choice to solve the optimization sub-problem of Eq. (5). Denote the solution to Eq. (5) by d_T, which is the horizontal part of the step direction vector. Accordingly, solving the normal and tangential sub-problems enables step direction, d_k, to be updated by Eq. (6).

$$d_k = d_N + d_T \tag{6}$$

All details of the computational requirements to estimate the step direction can be found in Scipy package of python programing [35] in the equality constraint optimization section. The main version of trust-region SQP method used a non- differentiable merit function to qualify the step direction and update the trust region in each step that required doing

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complicated calculations.

This study considers the merit function expressed as Eq. (7) known as the L1 merit function in the literature. In other word, the Armijo rule is replaced by modified calculation in each step.

$$\phi\left(x_k, c\right) = f\left(x_k\right) + c\left|G\left(x_k\right)\right| \tag{7}$$

The gradient of the merit function in Eq. (8) is used to control the reduction of the merit function in iterations.

$$\nabla\phi\left(x_k, c\right) = \nabla f\left(x_k\right) + c\left|\nabla G\left(x_k\right)\right| \tag{8}$$

The merit function is controlled by inequality in Eq. (9) to determine the appropriate step size and update the trust region.

$$\phi\left(x_{k+1}, c\right) - \phi\left(x_k, c\right) \le a.s_m.\left(\nabla\phi\left(x_k, c\right)^T d_k\right) \tag{9}$$

where a is a positive value equal to 0.5, and the parameter c is calculated by Eq. (10).

$$c = \gamma\left(\left\|x_k\right\| / \left\|\nabla G\left(x_k\right)\right\|\right) + \eta \tag{10}$$

where γ=2 and η=10 are positive values, and the parameter s_m is calculated by Eq. (11).

$$s_m = b^k \tag{11}$$

where b is a constant in the range of [0, 1], which is usually considered 0.5, and the parameter k shows the current iteration for determining the coefficient s_m. In the first step, this coefficient is considered equal to unity.

If Eq. (9) is stablished, s_m is accepted and the new trust radius can be increased to $7\times\|d_k\|$. This value is the enlargement radius used in the main version of the trust-region SQP that is properly approximated. Otherwise, this coefficient is reduced in each iteration by adding a unit to k up to the maximum number of predetermined iterations. Then, this coefficient is multiplied to trust radius and the calculations required for determining the step direction are repeated to obtain the convergence. In contrast to the traditional methods that perform the correction on the step size, this method applied the correction to the trust radius. Table 1 illustrates the full version of the algorithm as discussed in this section.

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Table 1: Member groups and corresponding allowable stresses for 25-bar truss

Choose a value for the parameters $\theta \in (0, 1)$ and $\xi \in (0, 1)$ and select stopping tolerance ε_{tol} and choose an initial value for x_0 and Δ_0 and set k=0.

While $k < k_{max}$:

Step 1: Compute Lagrange multiplier using BFGS method

Step 2: Compute the normal step d_N by the dogleg procedure

Step 3: Compute the normal step d_T by the PCG procedure

Step 4: Compute the total step $d = d_N + d_T$

Step 5: Check the merit function

If reduction is obtained: set $x_{k+1} = x_k + d_k$

Else: update Δ_k by Armijo rule and go to step 1

Step 6: If $\|\nabla f(x_k)+\lambda\nabla G(x_k)\| < \varepsilon_{tol}$: break the loop

Else: set $k = k + 1$

Compared to the original version, these changes aim to reduce the effect of the trust radius and the step direction obtained by SQP sub-problems. The second order correction, complicated trust radius updating, and reduction ratio of merit function are the instances excluded from the proposed method. Since the searched-based reliability analysis methods estimate the design point by the linearization of the limit state function, these changes have no undesirable effect on the final results and conversely lead to fast convergence.

3. PROBABILITY EXPECTATION

Recently, Rashki [36] presented a methodology based on methodic doubt to improve the accuracy of reliability analysis method that expresses a mathematically exact failure probability as Eq. (12). In another word, this method is a combination of the searched-based method and MCS with limited required samples.

$$P_f = \bar{P}_f + \xi \tag{12}$$

where P_f is the exact probability failure, $\bar{P}_f$ is the result of reliability analysis method as inaccurate failure probability, and x is the estimation error. The error term can be accurately estimated (x =0), overestimated (x <0), or underestimated (x >0) in different conditions. Eq. (12) could be expressed as Eq, (13) using expectation rules.

$$E\left(P_f\right) = E\left(\bar{P}_f\right) + E\left(\xi\right) = E\left(\bar{P}_f\right) + \mu_g \tag{13}$$

where m_g is the mean values of the error term. If the error estimator is unbiased, the term E($\bar{P}_f$) provides an accurate failure probability. Briefly, if the mean value of the error term is zero, the expectation of the failure probability obtained by the searched-based methods can be estimated by the Eq. (14).

$$E\left(\bar{P}_f\right)=E\left(\Phi\left(\beta_1\right)-\Phi\left(\beta_2\right)\right) \tag{14}$$

where b_1 and b_2 are the limit state function distances of each MCS sample from the mean. To find the solution of the Eq. (14), two problems need to be solved. The first is a simple optimization problem shown in Eq. (15).

$$\begin{cases} \min & \left\|x_k - c_1\alpha\right\| \\ \text{subject to} & G\left(x_k\right)=0 \end{cases} \tag{15}$$

where c_1 is the answer and a is the importance vector obtained by a searched-based method. It is the distance between the current position of a sample x_k and a point on the limit state surface (G=0). This distance is parallel to the line linked the origin to the design point obtained by the searched-based method in the standard normal space.

The second problem is a root-finding problem that intends to find the distance from modified position of the sample on the limit state surface to the line that intersects the origin and is parallel to the linearization of the limit state function at the design point obtained by searched-based method. This procedure is done for all MCS samples to find the new reliability indexes. Then, the mean is estimated as modified reliability index which is more accurate than the one estimated by the searched-based method. The root-finding problem is Eq. (16) which needs to be solved for c_2 parameter.

$$x_k+\left(c_1+c_2\right)\alpha=0 \tag{16}$$

The above-mentioned procedure is a simple one that can modify the accuracy of the final results. If this method fails to improve the accuracy, the alternative choice could be the Importance sampling that used the design point of the searched-based to generate new samples. The computational effort required for the Importance sampling is more compared to the probability expectation method, but sometimes it is necessary to implement it to overcome the time-consuming restrictions imposed by the Crude Monte Carlo simulation.

4. NUMERICAL SIMULATION

In this section, the numerical examples are investigated in the literature to evaluate the efficiency and performance of the proposed method. These examples include some challenging issues such as different nonlinearity types of limit state functions and complex numerical combinations of random variables. In order to simplify the process, the proposed method or trust-region SQP is denoted by TRSQP herein.

For each example, the origin coordinate of the standard normal space is selected as the

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initial point for different analysis algorithms. The result of the proposed method is compared to two states of art algorithms developed to overcome the high nonlinearity of the limit state function including DSTM and CFSL.

The setup of the DSTM and CFSL methods are the same as demonstrated in the literature [37,38], where the control factors are specified as 0.1 for DSTM, and the step length and adjusting coefficient of the CFSL method set 50 and 1.5, respectively. The same convergence criteria, i.e., $\|\nabla f(x_k)+\lambda\nabla G(x_k)\|$ are applied to all methods. The reliability index of the Monte Carlo simulation (MCS) computed by 1×10^6 samples is employed to qualify the accuracy of the methods.

In addition, the examples presented in this article can be solved in BI software which is a computer program for reliability analysis that is developed by the authors and can be downloaded from the (*www.betaindexsoftware.com*) and different examples can be modeled accordingly.

4.1 Example 1

The first example has a highly nonlinear quartic polynomial limit state function with two standard normal random variables as Eq. (17), which is extracted from the previous studies [37,39].

$$G(X) = X_1 - 1.7X_2 + 1.5(X_1 + 1.7X_2)^2 + 5 \tag{17}$$

The convergence histories of DSTM, CFSL, and TRSQP are shown in Fig. 1. Two methods (see Fig. 1) represent unstable reliability index though the behaviors differ. DSTM swings between two incorrect points which can be seen in the magnifier part of Fig. 1. CFSL shows the wrong convergence history and results in increasing the value of reliability index after some iterations. TRSQP represents the appropriate performance and can find the design point. The coordinate of the design point (-2.4408, 1.5264) that leads to reliability index is 2.8787. This value is the best result that can be expected from the linearization of the limit state function, where the minimum distance of the origin to the linear approximation of the limit state function is 2.8787. This problem could be solved by other methods such as iHLRF with 2852 function evaluation [37] and STM method with 302 function evaluations [37].

The next step is to improve the accuracy of the results. Fig. 2 shows the probability expectation procedure implemented to reduce the error of linearization. This problem consists of two directions. It can be employed in the random samples in probability expectation model. However, this study applied the Gaussian-Hermite method [40] with 9 points in each direction of standard normal space to generate initial samples, which is shown by the red circle in the Fig. 2. Then, the samples must be moved to the limit state function along the importance vector direction achieved by the TRSQP method. This movement is shown by the blue circle in the Fig. 2. The next step is to compute the error of each sample with respect to reliability index obtained by the TRSQP. The yellow line in Fig. 2 is the expanded line estimated by linearization of the limit state function at the design point. The light blue lines that link the samples on the limit state function to the linearized limit state function depicted the error of each sample. Finally, the improved result that applies the

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probability expectation (3.34) is in good agreement with MCS that resulted in the reliability index of 3.339.

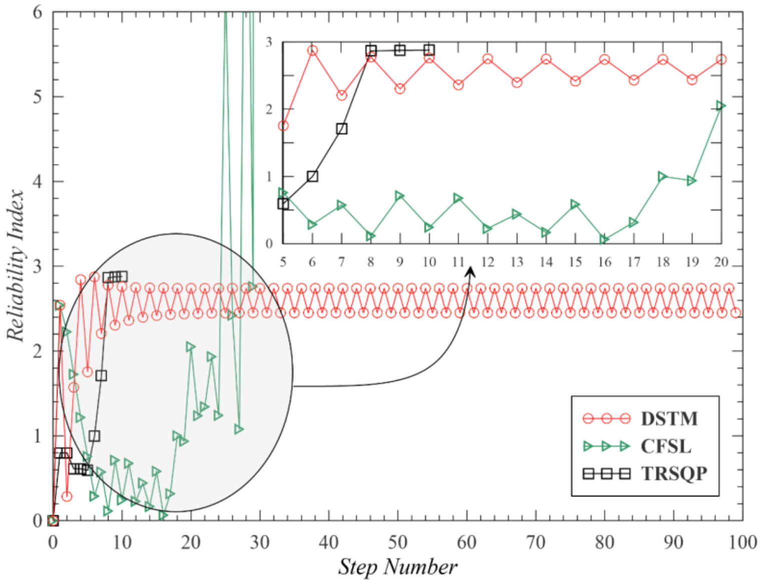

Figure 1. Iteration history for Example 1

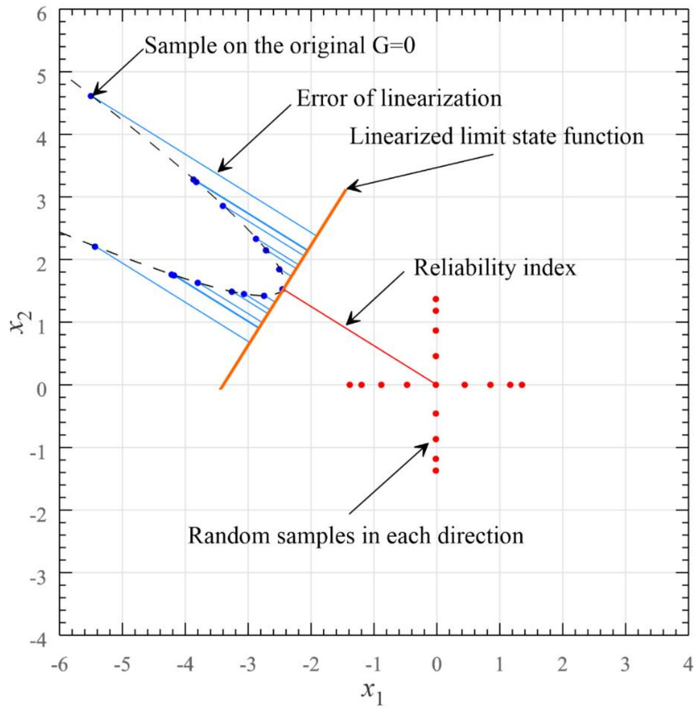

Figure 2. Sampling procedure to improve the accuracy of TRSQP (Example 1)

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Table 2: Results of various methods (Example 1)

Method	β	P_f	**Iterations**	**G-Evaluations**
DSTM	----	----	----	----
CFSL	----	----	----	----
TRSQP	2.878	0.002001	10	10
TRSQP-PE	3.340	0.000418	10+18	10+18
MCS	3.339	0.000420	----	10^6

Table 2 summarizes the final results of the method. TRSQP relates to the proposed method without probability expectation, but TRSQP-PE is the combination of the aforementioned proposed method and probability expectation method. The robust performance of the proposed method that could handle the nonlinearity of the limit state function and improve the accuracy of the final result along with low computational cost is depicted.

4.2 Example 2

A combination of exponential and logarithmic random variables is shown in Eq. (18) [18,38,41].

$$G(X) = \ln\left(e^{1+X_1-X_2}\right) + e^{5-5X_1-X_2} \tag{18}$$

Both random variables are standard normal random variables. Fig. 3 shows the iteration history of three methods with magnified view of the initial steps. The high nonlinearity of the limit state functions leads to fail convergence for DSTM and CFSL. DSTM oscillates between two wrong points and cannot resolve the problem by reliance on a stable solution. Although CFSL achieves the stable reliability index, this result is wrong and inaccurate even in the linearization phase. Without probability expectation, TRSQP acts as a fast convergent method.

Some methods such as the ones proposed by Roudak et al. [42], Gong and Yi [18], and Gong et al. [43] could potentially solve this problem. These methods handle the nonlinearity of the limit state function, but the computational efforts for applying these methods are not minimum. For better investigation, check the abovementioned references. Accordingly, it could be concluded that TRSQP is more robust than other methods which are deployed based on the first-order estimation.

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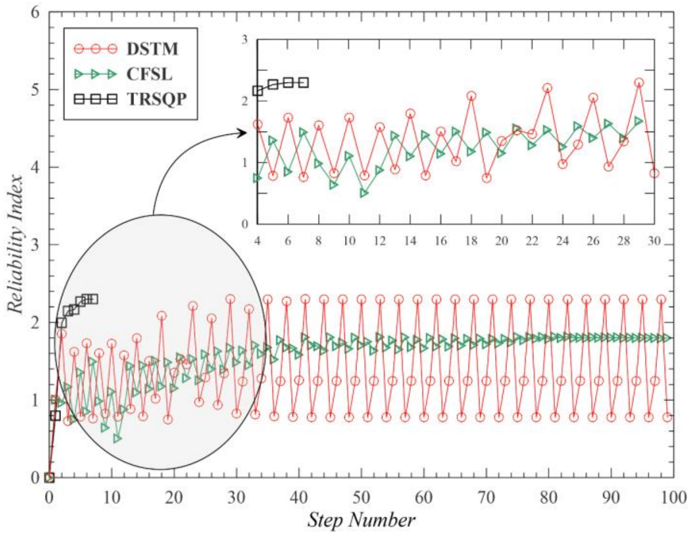

Figure 3. Iteration history for Example 2

The next step is to investigate the accuracy of the results. Similar to the Example 1, this example is a two-dimension case and the probability expectation could be implemented to modify the final results of the TRSQP. This time, fifteen points of Gaussian-Hermite method are implemented to find the error of linearization. Fig. 4 shows the position of the initial samples in each direction by the red circles. Then, the projections of these samples on the limit state function, along the importance vector of TRSQP method, are obtained as illustrated by the blue circles. Finally, the error of linearization is computed by the process mentioned in Section 3. The reliability index with error analysis (2.7) that is very close to the MCS output.

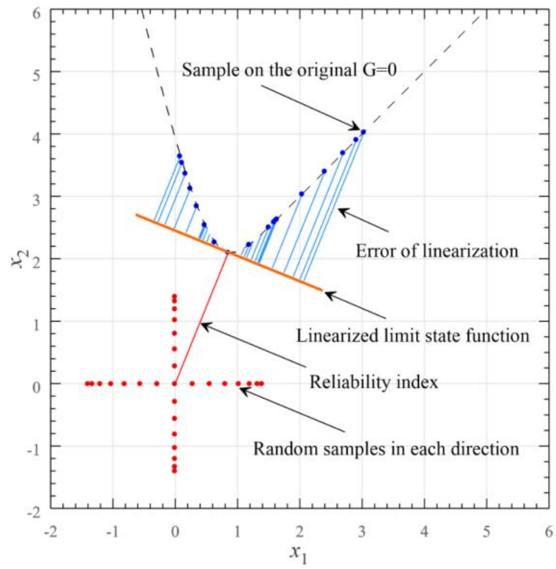

Figure 4. Sampling procedure to improve the accuracy of TRSQP (Example 2)

Tables 3 shows the results of the reliability index and the probability of failure in the last iteration. The results of TRSQP show that this method can overcome the high nonlinearity of the limit state function.

Table 3: Results of various methods (Example 2)

Method	β	P_f	Iterations	G-Evaluations
DSTM	----	----	----	----
CFSL	1.796	0.036247	100	100
TRSQP	2.299	0.010738	7	7
TRSQP-PE	2.700	0.003466	7+30	7+30
MCS	2.745	0.003025	----	10^6

This achievement is associated with an optimum computational cost that could be proven by comparing other methods in the literature. However, the accuracy of the final result needs further research, which is addressed in TRSQP-PE that has yielded significant result with only 30 samples.

4.3 Example 3

The limit state function with noise term is investigated in this Example [44]. Both random variables are the normal random variables with means of 1.5 and 2.5, respectively, and standard deviation of 1.0. The limit state function is shown in Eq. (19).

$$G(X)=\frac{(X_1^2+4)(X_2-1)}{20}-\sin\left(\frac{5X_1}{2}\right)-2 \tag{19}$$

Iteration history and the zoomed in view of initial iterations are shown in Fig. 5. Similar to the previous example, DSTM fails to converge and swing back and forth between three points. On the other hand, CFSL deviates from the convergence path and encounters the numerical stability issues. TRSQP searches the domain and reports the reliability index of 1.185 that is assumed to be the best solution in the linearization producer. However, there is a significant different between MCS reliability index 1.861 and TRSQP.

It is not a weakness of the TRSQP because it searches linearized space to find the shortest distance between origin and limit state surface and successfully locates the design point (0.44097, 1.10007) which can be seen in Fig. 6. The error relates to high nonlinearity of the limit state function which includes noise term. If simple probability expectation is implemented, no progress is made in error improvement because the limit state function experiences sever changes around the design point and linearized procedure of probability expectation cannot help. Then, this time, Importance sampling is applied to estimate the accuracy reliability index. Fig. 6 shows the Importance sampling procedures, where the design point is chosen as the central sample and the other one hundred samples are generated with a standard deviation of 0.2.

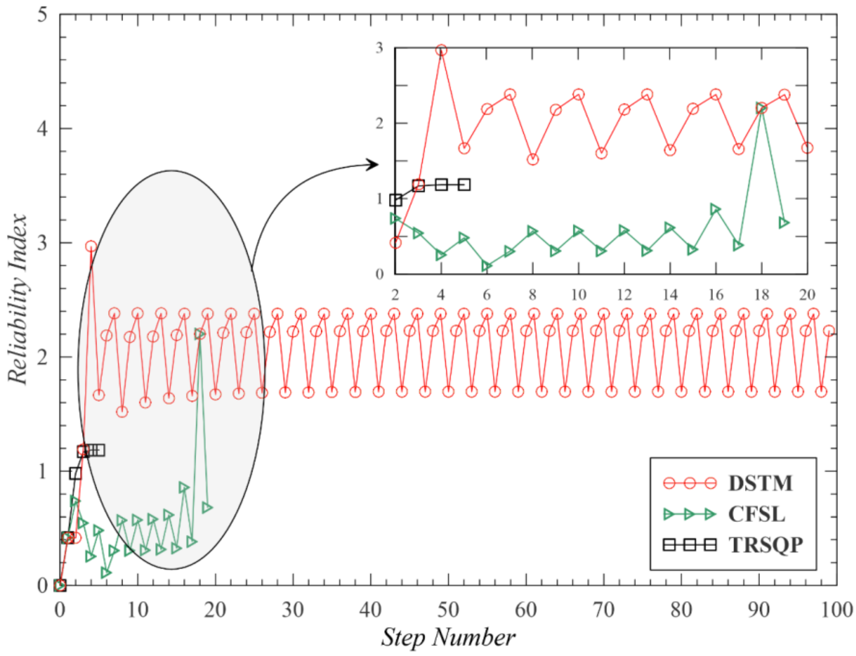

Figure 5. Iteration history for Example 3

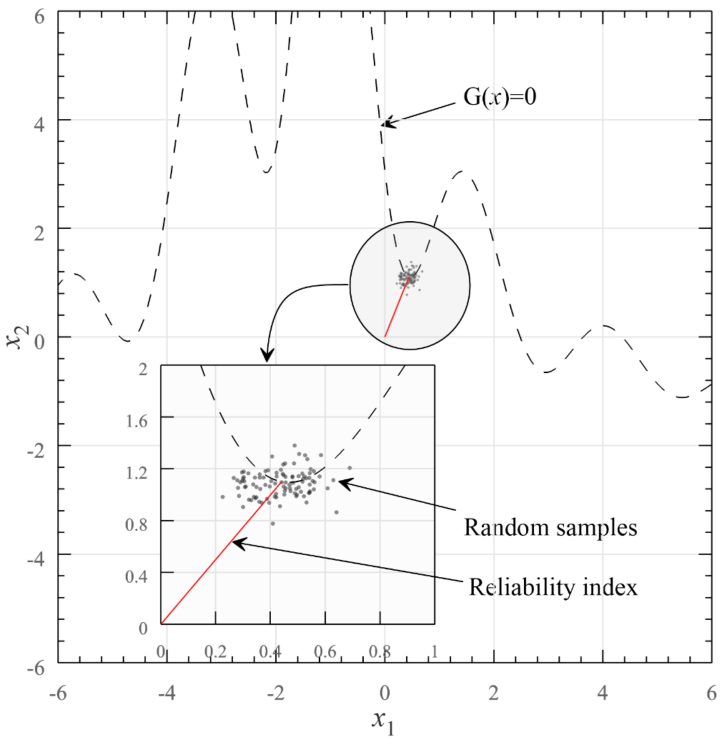

Figure 6. Sampling procedure to improve the accuracy of TRSQP (Example 3)

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TRSQP performs the best with 5 steps and the corresponding reliability index is the best answer that can be expected from a first-order method. The combination of TRSQP and Importance sampling, TRSQP-IS, leads to a proper output which is in good agreement with MCS. Table 4 shows the results of the methods including iteration number, number of function evaluation, reliability index, and failure probability.

Table 4: Results of various methods for Example 3

Method	$\boldsymbol{\beta}$	$\boldsymbol{P_f}$	**Iterations**	**G-Evaluations**
DSTM	----	----	----	----
CFSL	----	----	----	----
TRSQP	1.185	0.11797	5	5
TRSQP-IS	1.840	0.03179	5+100	5+100
MCS	1.861	0.03133	----	10^6

4.4 Example 4

Two-degree-of-freedom primary and secondary dynamic system with eight random variables are investigated as shown in Fig. 7.

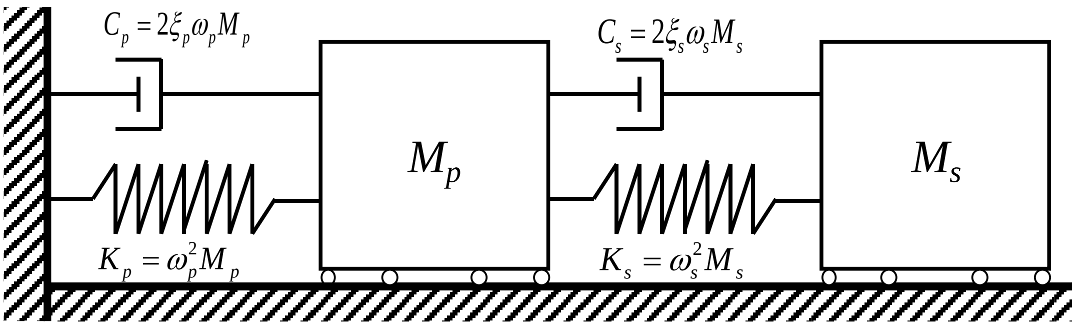

Figure 7. Primary secondary system (Example 4)

Table 5. Probability Distribution of Random Variable (Example 4)

Description	**Variable**	**Distribution**	**Mean**	**Standard deviation**
The primary mass	m_p	Lognormal	1	0.1
The secondary mass	m_s	Lognormal	0.01	0.001
The primary spring stiffness	k_p	Lognormal	1	0.2
The secondary spring stiffness	k_s	Lognormal	0.01	0.002
The primary damping ratio	ξ_p	Lognormal	0.05	0.02
The secondary damping ratio	ξ_s	Lognormal	0.02	0.01
The force capacity of the 2nd spring	F_S	Lognormal	15	1.5
The intensity of the white noise	S_0	Lognormal	100	10

Table 5 shows the statistical properties. The variables are the masses m_p and m_s, spring stiffnesses k_p and k_s, damping ratios ξ_p and ξ_s, the force capacity of secondary spring F_S, and the intensity of a white-noise base excitation of the system S_0. The subscripts p and s are the signs of primary and secondary oscillators, respectively. The limit state function of this example is defined by Eq. (20).

$$G(X) = F_S - k_s P\left(E\left[X_s\right]\right)^{\frac{1}{2}} \tag{20}$$

where P is the peak factor equal to 3. Parameter E can be expressed as Eq. (21).

$$E\left[X_s\right] = \frac{\pi S_0}{4\xi_s \omega_s^3}\left[\frac{\xi_a \xi_s}{\xi_p \xi_s \left(4\xi_a^2 + \theta^2\right) + \gamma \xi_a^2} \times \frac{\left(\xi_p \omega_p^3 + \xi_s \omega_s^3\right)\omega_p}{4\xi_a \omega_a^3}\right] \tag{21}$$

where $\gamma = m_s/m_p$ is the mass ratio, $\omega_p = (k_p/m_p)^{0.5}$ and $\omega_s = (k_s/m_s)^{0.5}$ define natural frequencies and damping ratios, respectively. $\omega_a = (\omega_p + \omega_s)/2$ represents the average frequency. $\xi_a = (\xi_p + \xi_s)/2$ specifies damping ratio and $\theta = (\omega_p - \omega_s)/\omega_a$ is the tuning parameter. The convergence histories of DSTM, CFSL, and TRSQP are shown in Fig. 8 with magnifier view of the initial steps. These three methods (see Fig. 8) represent stable reliability index though the behaviors are different. DSTM encounters some problems in convergence near the design point and CFSL illustrates the poor choice of step direction leading to non-uniform oscillation before convergence. TRSQP overcomes the weakness of the two mentioned method. In other words, proper step direction and fast convergence around the design point are the advantage of the TRSQP.

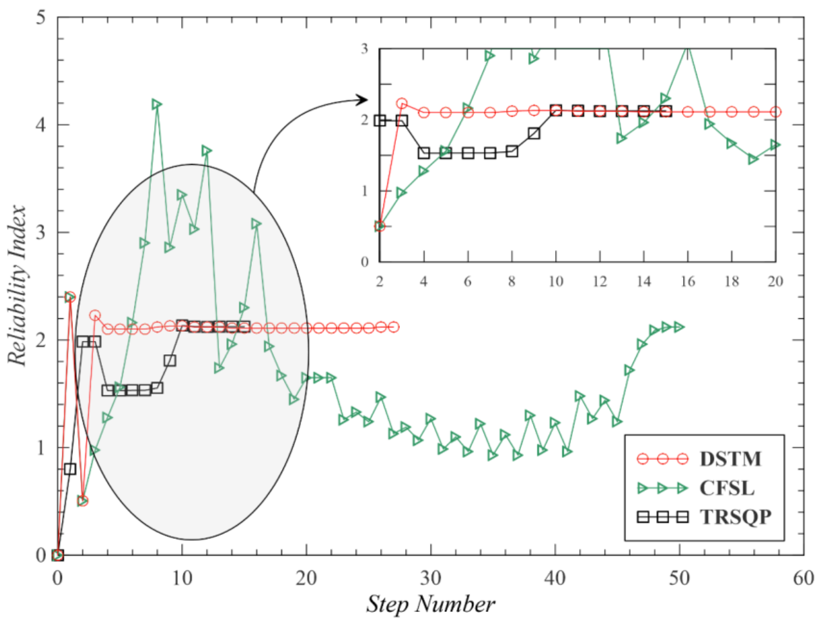

Figure 8. Iteration history for Example 4

Reliability analysis results for DSTM, CFSL, TRSQP, TRSQP-IS; and MCS with 10^6 samples are shown in Table 6. Similar to the previous problems, TRSQP successfully obtained the reliability index within a few iterations. DSTM and CFSL are ranked as the next ones with 27 and 54 iterations. The corresponding b= 2.12 is the best answer that can be expected from a first-order method. It is worth mentioning that if the Importance sampling method is applied to other methods, the same result could be obtained because this part of analysis does not need any prior information except for design point coordinates.

Table 6: Results of various methods (Example 4)

Method	β	P_f	Iterations	G-Evaluations
DSTM	2.1231	0.01687	27	27
CFSL	2.1231	0.01687	54	54
TRSQP	2.1230	0.01687	15	15
TRSQP-IS	2.7300	0.01687	15+2000	15+2000
MCS	2.7360	0.00310	----	10^6

5. DISCUSSION

In the previous section, the performance of the proposed method, which is an algorithm based on the trust-region sequential quadratic programming with new merit function (filter), namely TRSQP, was investigated by concentrating on four examples. These examples involved explicit limit state functions with different nonlinearity. The results of other reliability methods, including DSTM and CFSL which are state-of-art methods in the first-order category were shown for comparison. Furthermore, the response of the sampling method was estimated. First-order reliability methods are generally based on the steepest descent direction, where the priority is to reach the limit state surface. TRSQP uses a combination of dogleg and PCG to obtain a more appropriate step direction vector to continue iterations that address not only approaching the limit state surface, but also minimizing the optimization objective function.

In these examples, accuracy, robustness, and efficiency of the algorithms can be compared. Achieving the final stable result in first step, which is the linearized of limit state function, indicated the robustness of methods. The number of iterations and function evaluations are considered as the criteria of efficiency. The accuracy of each method can be investigated by checking the final result. The robustness and efficiency of the TRSQP is depicted in the examples, but the accuracy is a more challenging issue. Then, the combination of the TRSQP and a simple probability expectation or Importance sampling were applied to overcome the accuracy issue of the analysis. According to these definitions, the proposed method is superior to other first-order methods in all three items.

As shown in the tables, the reliability indexes obtained from the proposed method are close to the literature that demonstrates the robustness of the TRSQP. The minimum required steps and function evaluations of TRSQP to achieve the design point indicates the fast convergence that is the sign of efficiency. It is the most significantly competitive feature of the proposed method observed in Examples 1 to 4. The readers can test the proposed

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algorithm and other state-of-the-art methods in BI software understand the fast convergence of the proposed method compared to other in case of resolving challenging problems.

6. CONCLUSION

The first-order iterative algorithms are extensively employed to estimate the failure probability and design point in the reliability analysis in the linearized space. Due to the nonlinearity of the limit state surface, FORM-based algorithms fail to converge.

This paper proposes a reliability analysis method on the basis of the nonlinear optimization method with simple filter. The proposed method employs the appropriate tools to deal with the high nonlinearity challenges in case of resolving a reliability problem. TRSQP replaces the initial reliability problem with two optimization sub-problems that are easier to solve. Then, the dogleg and projected conjugate gradient methods are used to compute a step direction vector, which is different from the step direction of FORM-based methods. Subsequently, the step size is computed using a simplified merit function control and the first duty of the searched-based method to ensure the design point is done.

Through the application and test of several numerical and practical engineering examples in the literature, it could be concluded that TRSQP is a robust and efficient algorithm that could be used to resolve reliability problems. However, certain simple methods such as probability expectation method can be implemented to improve the accuracy of the algorithm such as MCS.

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